OCR
Oxford Cambridge and RSA

Tuesday 16 June 2015 – Afternoon

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(i)** Express $\dfrac{2}{3-x}+\dfrac{3}{1+x}$ as a single fraction in its simplest form. **[2]**

(ii) Hence express $\left(\dfrac{2}{3-x}+\dfrac{3}{1+x}\right)\times\dfrac{x^2+8x-33}{121-x^2}$ as a single fraction in its lowest terms. **[3]**

2 A triangle has vertices at A (1, 1, 3), B (5, 9, −5) and C (6, 5, −4). P is the point on AB such that $AP:PB=3:1$.

(i) Show that $\overrightarrow{CP}$ is perpendicular to $\overrightarrow{AB}$. **[4]**

(ii) Find the area of the triangle ABC. **[2]**

3 The equation of a curve is $y=e^{2x}\cos x$. Find $\dfrac{dy}{dx}$ and hence find the coordinates of any stationary points for which $-\pi\leqslant x\leqslant\pi$. Give your answers correct to 3 significant figures. **[6]**

4 **(i)** Find the first three terms in the binomial expansion of $(8-9x)^{\frac{2}{3}}$ in ascending powers of x. **[4]**

(ii) State the set of values of x for which this expansion is valid. **[1]**

5 By first using the substitution $t=\sqrt{x+1}$, find $\int e^{2\sqrt{x+1}}\,dx$. **[6]**

6 **(i)** Use the quotient rule to show that the derivative of $\dfrac{\cos x}{\sin x}$ is $\dfrac{-1}{\sin^2 x}$. **[2]**

(ii) Show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}\frac{\sqrt{1+\cos 2x}}{\sin x\sin 2x}\,dx=\frac{1}{2}(\sqrt{6}-\sqrt{2})$. **[6]**

7 A curve has equation $(x+y)^2=xy^2$. Find the gradient of the curve at the point where $x=1$. **[7]**

8 In the year 2000 the population density, P, of a village was 100 people per km^2, and was increasing at the rate of 1 person per km^2 per year. The rate of increase of the population density is thought to be inversely proportional to the size of the population density. The time in years after the year 2000 is denoted by t.

(i) Write down a differential equation to model this situation, and solve it to express P in terms of t. **[6]**

(ii) In 2008 the population density of the village was 108 people per km^2 and in 2013 it was 128 people per km^2. Determine how well the model fits these figures. **[2]**

9 Two lines have equations

$$\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k}) \text{ and } \mathbf{r} = 4\mathbf{i} + 10\mathbf{j} + 19\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} + \alpha\mathbf{k}),$$

where α is a constant.

Find the value of α in each of the following cases.

(i) The lines intersect at the point (7, 7, 1). **[3]**

(ii) The angle between their directions is 60°. **[4]**

10 **(i)** Express $\frac{x+8}{x(x+2)}$ in partial fractions. **[3]**

(ii) By first using division, express $\frac{7x^2+16x+16}{x(x+2)}$ in the form $P+\frac{Q}{x}+\frac{R}{x+2}$. **[3]**

A curve has parametric equations $x=\frac{2t}{1-t}$, $y=3t+\frac{4}{t}$.

(iii) Show that the cartesian equation of the curve is $y=\frac{7x^2+16x+16}{x(x+2)}$. **[4]**

(iv) Find the area of the region bounded by the curve, the x-axis and the lines $x=1$ and $x=2$. Give your answer in the form $L + M\ln 2 + N\ln 3$. **[4]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1	(i)		$\dfrac{2(1+x)+3(3-x)}{(3-x)(1+x)}$	B1	or $\dfrac{2(1+x)}{(3-x)(1+x)}+\dfrac{3(3-x)}{(3-x)(1+x)}$	allow recovery from omission of brackets; brackets may be expanded in numerator
			$\dfrac{11-x}{(3-x)(1+x)}$ oe isw	B1	numerator must be simplified **B2** if unsupported	denominator may be in expanded form at either stage eg $3+2x-x^2$
				[2]		
1	(ii)		$\dfrac{(x+11)(x-3)}{(11+x)(11-x)}$ or $\dfrac{(x+11)(x-3)}{(121-x^2)}$	M1*	allow $(x-11)(x+3)$ for numerator and / or $(x-11)(x+11)$ in denominator	
			their $\dfrac{11-x}{(3-x)(1+x)}\times$ their $\dfrac{(x+11)(x-3)}{(11+x)(11-x)}$	M1*dep	or $\dfrac{2}{(3-x)}\times$ their $\dfrac{(x+11)(x-3)}{(11+x)(11-x)}$ $+\dfrac{3}{(1+x)}\times$ their $\dfrac{(x+11)(x-3)}{(11+x)(11-x)}$	with at least one pair of their terms correctly cancelled out, allow if RH fraction only partially factorised
			$\dfrac{-1}{(1+x)}$ oe cao	A1		
				[3]		

Question			Answer	Marks	Guidance	
2	(i)		$k(4\mathbf{i} + 8\mathbf{j} - 8\mathbf{k})$ oe seen	B1	allow eg (4, 8, − 8)	NB $\overrightarrow{AB} = 4\mathbf{i} + 8\mathbf{j} - 8\mathbf{k}$ $\overrightarrow{AP} = 3\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$ $\overrightarrow{BP} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$
			$[\overrightarrow{CP} =] -2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ or $[\overrightarrow{PC} =]\ 2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$	B1	allow eg (−2, 2, 1)	NB P is (4, 7, − 3)
			their $\overrightarrow{CP}$. their $k(4\mathbf{i} + 8\mathbf{j} - 8\mathbf{k})$ evaluated	M1	allow one numerical error	
			$\overrightarrow{AB}\cdot\overrightarrow{CP} = 4\times(-2) + 2\times8 + 1\times(-8) = 0$ oe	A1 [4]	arithmetic must be shown and be consistent	if 0, allow B4 for fully correct solution using eg Pythagoras or trigonometry
2	(ii)		½ × their AB × their CP oe	M1	or ½$ab\sin C$ from triangle ABC with a, b and C correct	$AB = \sqrt{4^2 + 8^2 + (-8)^2} = 12$ $CP = \sqrt{(-2)^2 + 2^2 + 1^2} = 3$ $AC = \sqrt{5^2 + 4^2 + (-7)^2} = \sqrt{90}$ $BC = \sqrt{1^2 + (-4)^2 + 1^2} = \sqrt{18}$ $AP = 9$ and $BP = 3$ $A = 18.4°$, $B = 45°$ and $C = 116.6°$
			18	A1 [2]		

Question			Answer	Marks	Guidance	
3			$\frac{dy}{dx} = \pm k e^{2x} \cos x \pm e^{2x} \sin x$	**M1***	k is any constant	Product Rule
			$\frac{dy}{dx} = 2e^{2x} \cos x - e^{2x} \sin x$ oe	**A1**		
			their $\frac{dy}{dx} = 0$	**M1dep***		
			$\tan x = 2$ or $\cos x = (\pm)\frac{1}{\sqrt{5}}$ or $\sin x = (\pm)\frac{2}{\sqrt{5}}$	**A1**	ignore omission of "$e^{2x} = 0$ has no solution",	or $\sqrt{5}\cos(x + \tan^{-1}½) = 0$
			$x = 1.11$ and -2.03 cao	**A1**	(1.11, 4.09) and / or (– 2.03 , – 0.00765)	if **A0A0**, **SC1** for all 4 values to greater precision 1.107…, – 2.034…, 4.094…., – 0.0076457…(or – 0.007646)
			$y = 4.09$ and -0.00765 cao	**A1**	or **A1** for each correct pair of co-ordinates: mark to benefit of candidate	NB $x = 1.107148718$ and -2.034443936 $y = 4.094229238$ and -0.007645738
					extra values within range incur a penalty of one mark; **or** any finite value for x obtained from $e^{2x} = 0$ incurs a penalty of one mark	ignore extra values outside range
				[6]		

Question			Answer	Marks	Guidance	
4	(i)		$8^{2/3}=4$	B1		may be embedded
			$(1-\frac{9x}{8})^{2/3}$ seen	M1	$8^{2/3}+(2/3)8^{-1/3}(\pm 9x)+\frac{2/3\times(2/3-1)}{2!}8^{-4/3}(\pm 9x)^2$	ignore extra terms
			$1+\left(\frac{2}{3}\right)\left(\frac{\pm 9x}{k}\right)+\frac{1}{2!}\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)\left(\frac{\pm 9x}{k}\right)^2$ where k is an integer greater than 1	M1	$4+(2/3)(\frac{1}{2})(\pm 9x)+\frac{2/3\times(2/3-1)}{2!}(\frac{1}{16})(\pm 9x)^2$	or better
			$4-3x-\frac{9}{16}x^2$ or $4(1-\frac{3}{4}x-\frac{9}{64}x^2)$ cao	A1		
				[4]		
4	(ii)		$-\frac{8}{9}<x<\frac{8}{9}$ or $\lvert x\rvert<8/9$ isw cao	B1		
				[1]		

Question			Answer	Marks	Guidance	
5			$\frac{dt}{dx} = k(x+1)^{-\frac{1}{2}}$ or $\frac{dx}{dt} = 2t$ from $x = t^2 \pm 1$ oe	M1	or eg $k\,dt = \frac{dx}{\sqrt{x+1}}$ oe	
			$\int kte^{2t}\,dt$	M1*	k is any non-zero constant	
			$kt \times \frac{1}{2}e^{2t} \pm k\int \frac{1}{2}e^{2t}\,dt$	M1dep*		
			$te^{2t} - \int e^{2t}\,dt$	A1	may be implied by the next **A1**	
			$te^{2t} - \frac{1}{2}e^{2t}$	A1		
			$\sqrt{x+1}e^{2\sqrt{x+1}} - \frac{1}{2}e^{2\sqrt{x+1}} + c$ cao www	A1	$+ c$ may be seen in previous line only for **A1**	if dt is not seen in the integral at some point impose a penalty of 1 mark from total mark of 2 or more
				[6]		
6	(i)		$\frac{\sin x \times -\sin x - \cos x \times \cos x}{\sin^2 x}$ may be implied by $\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$	M1	or $-\sin x \times \frac{1}{\sin x} + \cos x \times -(\sin x)^{-2} \times \cos x$ oe	allow sign errors only if **M0**, **SC1** for just $\frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x}$
			eg $= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$ and completion to $\frac{-1}{\sin^2 x}$ **AG**	A1	eg $= \frac{-\sin^2 x}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}$ oe and completion to $\frac{-1}{\sin^2 x}$	need to see at least two correct, constructive steps and statement of given answer for **A1** NB $\sin^2 x + \cos^2 x = 1$ seen may be a constructive intermediate step
				[2]		

Question			Answer	Marks	Guidance	
6	(ii)		$\cos 2x = 2\cos^2 x - 1$ substituted in numerator	M1	or alternative form of double angle formula plus Pythagoras leading to no term in $\sin^2 x$ in numerator	may be awarded if not seen as part of fraction
			$\sin 2x = 2\sin x\cos x$ substituted in denominator	M1		
			$\dfrac{\sqrt{2}\cos x}{2\sin^2 x\cos x}$	A1		NB $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \frac{1}{\sqrt{2}\sin^2 x}\,dx$
			$F[x] = \pm k\dfrac{\cos x}{\sin x}$	M1*	k must not be obtained from square rooting a negative number	NB $-\dfrac{\cos x}{\sqrt{2}\sin x}$
			$F\left[\dfrac{1}{4}\pi\right] - F\left[\dfrac{1}{6}\pi\right]$	M1dep*	eg $\dfrac{-\cos \pi/4}{\sqrt{2}\times\sin \pi/4} - \dfrac{-\cos \pi/6}{\sqrt{2}\times\sin \pi/6}$	eg $\dfrac{-1/\sqrt{2}}{\sqrt{2}\times 1/\sqrt{2}} - \dfrac{-\sqrt{3}/2}{\sqrt{2}\times 1/2}$
			$= \dfrac{1}{2}(\sqrt{6}-\sqrt{2})$ www **AG**	A1		at least one correct intermediate step following substitution needed as well as statement of given result eg $-\dfrac{\sqrt{2}}{2}(1-\sqrt{3})$
				[6]		

Question			Answer	Marks	Guidance	
7			LHS is $k(x+y)(1+\frac{\mathrm{d}y}{\mathrm{d}x})$	M1	or $2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}+ky+kx\frac{\mathrm{d}y}{\mathrm{d}x}$ k is any positive integer	some terms may appear on RHS with signs reversed
			$k = 2$	A1		if **M0** in middle scheme, **SC1** for **three terms out of four** completely correct with $k = 2$
			$2y\frac{\mathrm{d}y}{\mathrm{d}x}$ on RHS from differentiating y^2	B1		may appear on LHS with sign reversed
			$y^2+Kxy\frac{\mathrm{d}y}{\mathrm{d}x}$ on RHS	M1	K is any positive integer	NB $K = 2$; may appear on LHS with signs reversed
			obtains a value of y from eg $(1+y)^2 = 1\times y^2$ oe	M1	allow even if follows incorrect manipulation	NB $y = -0.5$
			substitution of $x = 1$ and their y dependent on at least two correct terms seen following differentiation, even if follows subsequent incorrect manipulation	M1	may be implied by $1+\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{4}-\frac{\mathrm{d}y}{\mathrm{d}x}$	or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2-1-0.25}{-1-2+1}$ NB $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x+2y-y^2}{2xy-2x-2y}$
			$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3}{8}$ oe cao	A1 [7]		-0.375

Question			Answer	Marks	Guidance	
8	(i)		$\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{k}{P}$	B1	or $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{1}{kP}$	k should be unspecified at this stage
			$k = 100$ from $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{k}{P}$	B1	or $k = 0.01$ from $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{1}{kP}$	may be seen later
			$\int P\mathrm{d}P=\int(\text{their } k)\mathrm{d}t$	M1*	allow $k = 1$	allow omission of $\int$ and recovery of omission of one operator for **M1*A1**
			$\frac{P^2}{2}=kt+c$	A1	or $t=\frac{P^2}{2k}+d$	if **M0**, **SC2** for $\ln P = kt + c$ thereafter only **M1** may be earned
			substitution of $t = 0$ and $P = 100$	M1dep*	may follow incorrect algebraic manipulation, but equation must include c (or d)	NB $c = 5000$ or $d = -50$
			$P=\sqrt{10000+200t}$ or $10\sqrt{100+2t}$ or $P=\sqrt{200(50+t)}$ isw cao	A1 [6]		allow recovery from eg use of x for P throughout, but withhold final **A1** for eg $x=\sqrt{10000+200t}$
8	(ii)		$t = 8$, $P = 107.7$ or 108 so model was a good fit in 2008 oe	B1	or $t = 8.3(2)$ when $P = 108$ + comment	value of P or t must be found and correct comment made in each case; comments may be in same sentence.
			$t = 13$, $P = 112(.2)$, so model was not appropriate in 2013 oe	B1 [2]	or $t = 31.9(2)$ or 32 when $P = 128$ + comment comments may be in same sentence, but both values must be referenced	if **B0B0**, **SC1** for both values found no FT marks available comments on trends, extrapolation etc do not score just ticks / crosses etc do not score

Question			Answer	Marks	Guidance	
9	(i)		$\mu = 3$ soi	**B1**		from $3 + 2\lambda = 4 + \mu$ and $5 + \lambda = 10 - \mu$ NB $\lambda = 2$
			$1 = 19 + (\text{their } 3)\times\alpha$ oe	**M1**		do not allow sign errors
			$[\alpha =]-6$	**A1**		
				[3]		
9	(ii)		$2\times1+1\times(-1) + 1\times\alpha$	**M1***	allow 1 sign error	NB $1+\alpha = \sqrt{6}\times\sqrt{(2+\alpha^2)}\times\cos60°$
			$\sqrt{(2^2+1^2+1^2)}\times\sqrt{(1^2+(-1)^2+\alpha^2)}\times\cos60°$	**M1***	allow 1 slip, eg sign error or omission of power	
			eg their $4 + 8\alpha + 4\alpha^2 = 6(2+\alpha^2)$	**M1dep***	square both sides	
			$\alpha = 2$ cao	**A1**	if **M1M1M0, B2** for unsupported or alternative valid method	NB $2\alpha^2 - 8\alpha + 8 = 0$
				[4]		
10	(i)		$\frac{A}{x}+\frac{B}{x+2}$	**B1**		award if only implied by answer
			$x + 8 = A(x+2) + Bx$ soi	**M1**	allow one sign error	clearing fractions successfully
			$A = 4$ *and* $B = -3$	**A1**		if **M0**, **B1** for each value www
				[3]		

Question			Answer	Marks	Guidance	
10	(ii)		quotient (P) is 7	**B1**		
			$2x + 16$ seen	**B1**	if **B0**, **B1** for Q =8 and **B1** for $R = -6$ www	eg as remainder or in division chunking
			$7 + \frac{8}{x} - \frac{6}{x+2}$	**B1**		or allow $P = 7$, $Q = 8$ $R = -6$
				[3]		
10	(iii)		$t = \mathrm{f}(x)$	**M1***	from $x = \frac{2t}{1-t}$; **M0** for $t = \mathrm{g}(y)$	
			$t = \frac{x}{x+2}$	**A1**	or **B2** if unsupported	
			$y = 3 \times \text{their} \frac{x}{x+2} + \frac{4}{\text{their} \frac{x}{x+2}}$	**M1dep***		
			eg $\frac{3x^2 + (8+4x)(x+2)}{x(x+2)}$ and completion to			at least one correct, constructive, intermediate step shown
			$y = \frac{7x^2 + 16x + 16}{x(x+2)}$ www **AG**	**A1**		if **M0M0**, **SC2** for substitution of $x = \frac{2t}{1-t}$ in RHS of given equation and completion with at least two correct, constructive intermediate steps to $y = 3t + \frac{4}{t}$ www
				[4]		

Question			Answer	Marks	Guidance	
10	(iv)		$\int$ their $(P+\frac{Q}{x}+\frac{R}{x+2})$[d$x$]	**M1***	where P, Q and R are constants obtained in (ii)	allow omission of dx
			$\mathrm{F}[x]=7x+8\ln x-6\ln(x+2)$	**A1FT**	allow recovery from omission of brackets in subsequent working	if **M0**, **SC1** for $Px+Q\ln x+R\ln(x+2)$ where constants are unspecified or arbitrary
			$\mathrm{F}[2]-\mathrm{F}[1]$	**M1dep***		
			$7-4\ln 2+6\ln 3$	**A1** [4]		